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LETTER TO THE EDITOR

Multiscaling of cluster-mass distribution in a simple model of cluster–cluster aggregation with injection

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Abstract. The multiscaling structure of the cluster-mass distribution is investigated in a simple cluster–cluster aggregation model with injection in which the volume (or size) of clusters is infinitesimal but the mass of clusters is finite. The model is consistent with the Scheidegger river network model. It is shown that the partition function $Z(q) \equiv \sum_{n=1}^{N} M(t,n)^q$ scales as $Z(q) \approx t^{\zeta(q)}$ where $M(t,n)$ is the mass of the cluster on the site n at the time t and the summation ranges over all sites. In the limit of a sufficiently large q, $\zeta(q)/q$ (or $\partial\zeta(q)/\partial q$) gives the exponent γ of growth of a typical cluster. The exponent also equals to the fractal dimension $d_f = 1.5$ of a typical river in the Scheidegger river network model. The f–α spectrum of the normalized mass distribution is derived. It is found that the growth exponent of a typical cluster is exactly given by $\gamma = 1 - \alpha(\infty)$. The multiscaling of the cluster-mass distribution has a characteristic property for the simple cluster–cluster aggregation system.

Recently, there has been increasing interest in fractal structures of growth processes such as the cluster–cluster aggregation (CCA) model, the diffusion-limited aggregation (DLA) model, the ballistic deposition model and the river network model [1–11]. The CCA model presents the prototype of colloidal aggregation, smoke aggregation and droplet coalescence [9]. In the CCA model, there is the dynamic scaling of the cluster-size distribution [12]. The DLA model presents a prototype of the pattern formation of diffusive systems including electrodeposition, crystal growth, viscous fingering and bacterial colonies [13]. The ballistic deposition model provides a basis for understanding deposition processes used to prepare a wide variety of thin-film devices [14]. Branched river networks are among nature's most common patterns, spontaneously producing fractal structure [11, 15]. Some models have been constructed for the evolution of an entire drainage network [15, 16].

Very recently, the multifractal properties of the DLA have attracted considerable attention [17]. It has become clear that the DLA aggregate cannot be fully characterized by its fractal dimensionality. In order to characterize the aggregate further, it is necessary to derive the multifractal structure of the growth probability distribution. From the multifractality, one can obtain detailed information on the capability of each perimeter site to grow and, therefore, more information on the surface structure [17–22].

In this letter, we investigate the scaling structure of the cluster-mass distribution in a simple cluster–cluster aggregation model with injection. In the CCA model, the volume (or size) of clusters is infinitesimal but the mass of clusters is finite. The model is equivalent to the Scheidegger river network model [23]. It has been known that the cumulative cluster-mass distribution $P(\geq M)$ shows the following power-law asymptotic distribution:

$$P(\geq M) \approx M^{-1/3} \tag{1}$$

where M indicates the mass of a cluster [23]. The power-law distribution of the cluster-mass distribution is satisfied for a sufficiently large mass M. However, we will show that the cluster-mass distribution has the multiscaling structure.

First, we introduce the simple CCA model with injection [23]. The clusters with integer mass with an infinitesimal volume (or size) are placed on each site of the one-dimensional lattice and they coalesce by random-walk processes with discrete time steps. The time evolution of the model is defined by the following procedure: at the beginning of each time step, there is a single particle on every site of the lattice. All of them independently jump to nearest-neighbour sites according to the given probability $\frac{1}{2}$. When two clusters collide at a site after the jump, they coalesce to form a new cluster with a conserved mass. Then, a single particle with unit mass is added to every site. Thus the evolution of one time step is completed and we repeat this procedure. As a result, there is non-zero integer mass on every site at every time. The mass $M(t, n)$ of the cluster on the nth site at time t satisfies the stochastic equation

$$M(t+1, n) = w(t, n, n)M(t, n) + w(t, n+1, n)M(t, n+1) + 1 \tag{2}$$

where $w(t, n, m)$ denotes the realization that the cluster on the nth site jumps to the mth site at time t and is given by

$$w(t, n, m) = \begin{cases} 1 & \text{with probability } \frac{1}{2} \\ 0 & \text{with probability } \frac{1}{2}. \end{cases} \tag{3}$$

We perform the computer simulation of the simple CCA model for the one-dimensional lattice of $N = 100$–500. By the use of (2), the mass of cluster on each site is calculated under a periodic lateral boundary condition. Figure 1 shows the typical cluster-mass distributions for $t = 20$, 30 and 40 (the number of sites $N = 100$). The partition function $Z(q)$ is defined as the moments of the cluster mass

$$Z(q) \equiv \sum_{n=1}^{N} M(t, n)^q \tag{4}$$

where the summation ranges over all sites on the one-dimensional lattice. We study the scaling behaviour of the partition function (4). Figure 2 shows the log–log plot of the moments against the time t. It is confirmed that for sufficiently large t the partition function scales with time t as

$$Z(q) \approx t^{\zeta(q)}. \tag{5}$$

Figure 3 shows the $\zeta(q)$ behaviour against q. The scaling behaviour cannot be characterized by a single gap scaling. Since the total number of clusters equals to the size N of the lattice, the exponent $\zeta(0)$ gives $\zeta(0) = 0$. If $q < 0$, $\zeta(q) < 0$. If $q > 0$, $\zeta(q) > 0$. The exponent $\zeta(1)$ gives $\zeta(1) = 1$ since the sum of mass over all clusters is proportional to the time t. For a sufficiently large q, $\zeta(q)/q$ or $\partial\zeta(q)/\partial q$ gives the scaling exponent of largest growth rate of a cluster. It represents the scaling exponent γ of growth of a typical cluster. The exponent γ, is given by $\gamma = 1.50 \pm 0.02$. $\zeta(q)/q$ is related to the exponent of the growth rate. The growth rate of the cluster increases with the cluster mass. The origin of the multiscaling (5) is due to the increase of the growth rate with the cluster mass. The coalescence process between clusters may be represented by an underlying multiplicative process.

In order to characterize the multifractality of the cluster-mass distribution, it is convenient to normalize the cluster mass. We define the normalized partition

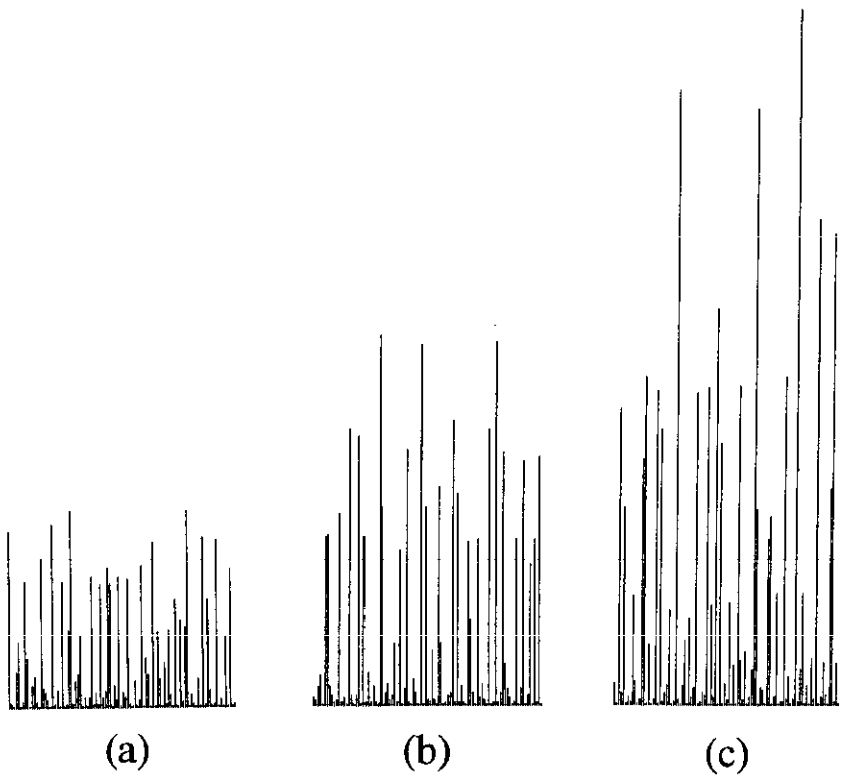

Figure 1. The typical cluster-mass distributions generated by the simple CCA model with injection. This run was done in a one-dimensional lattice $N = 100$ under a periodic lateral boundary condition for an illustration. The height of lines is proportional to the cluster mass. The cluster-mass distribution on the one-dimensional lattice at the time (*a*) $t = 20$, (*b*) $t = 30$ and (*c*) $t = 40$.

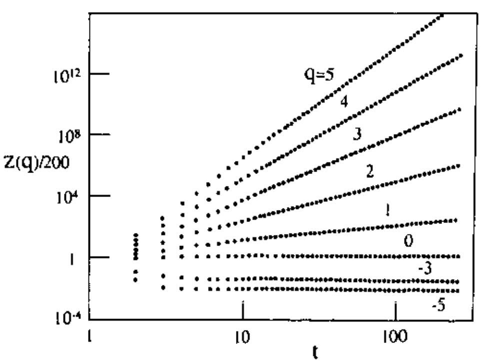

Figure 2. The log–log plot of the moments (4) against time t showing scaling behaviour.

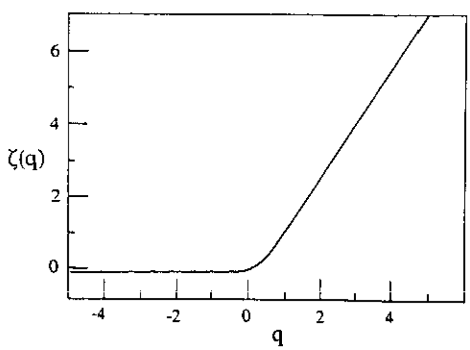

Figure 3. The behaviour of the scaling exponent $\zeta(q)$ against q.

function $Z_0(q)$

$$Z_0(q) \equiv \left\{ \sum_{n=1}^{N} M(t,n)^q \right\} \left\{ \sum_{n=1}^{N} M(t,n) \right\}^{-q} = Z(q)/\{Z(1)\}^q. \tag{6}$$

For a sufficiently large t, the normalized partition function scales as

$$Z_0(q) \approx t^{-\tau(q)}. \tag{7}$$

Figure 4 shows the plot of exponent $\tau(q)$ against q. With the Legendre transformation of $\tau(q)$, we obtain the f-α spectrum

$$f(q) = q\alpha(q) - \tau(q) \tag{8}$$

where $\alpha(q) = \partial\tau(q)/\partial q$ is the variable conjugate to q. Figure 5 shows the f-α spectrum. The f-α spectrum has a characteristic property of the cluster-mass distribution in the simple CCA model. The maximum value $f(0)$ of $f(\alpha)$ is given by

$$f(0) = \zeta(0) = 0. \tag{9}$$

The maximum value of α gives the scaling exponent of the minimum fraction of the cluster mass. The minimum value of α gives the scaling exponent of the maximum fraction of the cluster mass. The minimum value $\alpha(\infty)$ is exactly related to the growth exponent γ of a typical cluster

$$\alpha(\infty) = (\partial\tau/\partial q)_{q=\infty} = -\left[\frac{\partial}{\partial q} \left\{ \frac{\ln Z(q)}{\ln t} \right\} \right]_{q=\infty} + \frac{\ln Z(1)}{\ln t} = 1 - \gamma \tag{10}$$

where $\ln Z(1)/\ln t = 1$. The minimum value $\alpha(\infty)$ obtained from the simulation is given by 0.49 ± 0.02. We obtain the growth exponent $\gamma = 1.51 \pm 0.2$ from (10). This value is consistent with 1.50 obtained by the direct simulation [23]. The properties of the CCA should be characterized by the infinite set of exponents or the f-α spectrum.

In summary, we found the multiscaling structure of the cluster-mass distribution in the simple CCA model. We derived the f-α spectrum of the normalized mass distribution. We showed that the growth exponent γ was exactly given by $\gamma = 1 - \alpha(\infty)$.

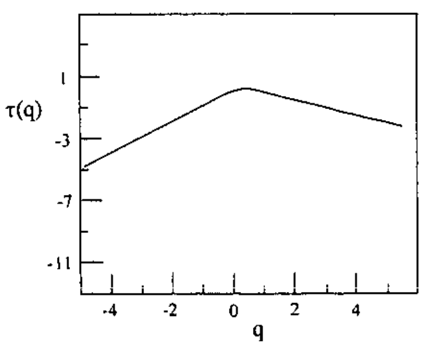

Figure 4. The plots of the exponents $\tau(q)$ against q.

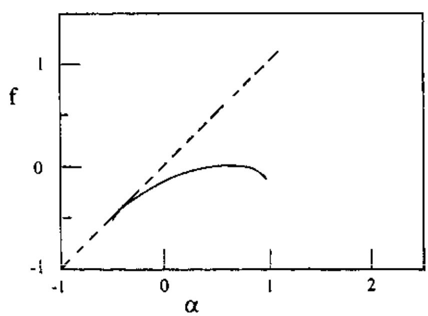

Figure 5. The f-α spectrum of the cluster-mass distribution.

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